Appendix - Estimation of σ_a

The half-normal distribution is specified by the probability density function

$$f(BMI^*) = 2\frac{1}{\sqrt{2\pi}\sigma_a} e^{-(BMI^*)^2/(2\sigma_a^2)}, BMI^* \geq 0,$$

with shape parameter σ_a. $BMI^* = (BMI - BMI_{95th_a})$ where BMI_{95th_a} is the BMI at a given sex- and age-specific 95th percentile at 6-month age interval a. Two important properties of the half-normal distribution are:

P1: $E(BMI^*) = \sqrt{\frac{2}{\pi}}\sigma_a$, and

P2: $E(BMI^{*2}) = \sigma_a^2$.

Based on relation P1 above, a finite population version of σ_a can be considered as a parameter representing the population mean of a hypothesized finite population. This population parameter has form

$\sqrt{\frac{\pi}{2}} \frac{\sum_{i=1}^{N_a} BMI_{ai}}{N_a}$, where the population has N total units, with class totals N_a and BMI_{ai}^* the individual's excess BMI above the 95th population percentile. The standard complex-survey weighted estimator is

$$\sigma_a = \sqrt{\frac{\pi}{2}} \frac{\sum_{i=1}^{n_a} w_i BMI_i^*}{\sum_{i=1}^{n_a} w_i},$$

where a is the sex-specific 6-month age interval, n_a is the sample size in interval a, and, w_i is the NHANES survey weight for individual i. This functional form is considered as an approximately unbiased estimator for the population mean. Relation P2 leads to an estimator of similar form for σ_a^2, but a required square root may lead to bias. After practical trials, an estimator based on property P1 appeared to provide the best fit with the data.