Supplemental Appendix

MODELING ASSUMPTIONS AND SENSITIVITY ASSESSMENTS

In this supplement, we provide a description of the assumptions underlying the final models used to produce the estimates of impact rate and linear acceleration (LA) presented in the accompanying paper. In addition, we outline the methods used to derive the estimates and present sensitivity analyses in which the final estimates are compared to those derived under alternative modeling assumptions.

For all models considered, we define an athletic exposure (AE) as any event, either a practice or a game, during which a player is at risk for an impact.

Preliminary analyses of the data led to identification of five players, three tackle and two flag, with total impact counts of magnitude 10 g or larger that appeared implausible. These players were identified using an estimated cutoff value C, where

$$C = Q_3 + 2.5 * IQR$$

with Q_3 equal to the 3rd quartile of the observed impact counts and IQR is the interquartile range. Values for Q_3 and IQR were calculated separately for tackle and flag players resulting in separate values of C for each football type. Based on this analysis, data for these five players were excluded from the primary analyses. As a result, unless otherwise specified, the data set on which these results are based contains 524 players.

MODELING ASSUMPTIONS

Estimation of Rates for Impacts ≥ 10 g

Let I_{ij} be the total number of impacts, summed across all AEs, of 10 g magnitude or higher incurred by player i on team j. We assume that I_{ij} is a sample from a Poisson distribution such that

$$I_{ij} \sim Poisson\left(AE_{ij}\, \lambda_{ij} \right) \qquad [1]$$

where

AE_{ij} = the number of AEs attended by player ij, and

λ_{ij} = the rate of impacts per AE experienced by player ij.

In addition, assume that the impact rate per AE can be modeled as

$$\ln(\lambda_{ij}) = \ln(rate_{flag}) + \ln(rr_{tackle}) * type_j + u_{ij} \qquad [2]$$

where

$rate_{flag}$ = the impact rate per AE among flag players,

rr_{tackle} = the ratio of the tackle to flag impact rates,

$type_j$ = 0 if team j is a flag team and 1 if it is tackle, and

u_{ij} = a player-specific random effect such that $u_{ij} \sim N(0, \gamma^2)$.

Note that, using the parameterization in equation [2], the natural log of the impact rate among tackle players can be estimated as

$$\ln(rate_{tackle}) = \ln(rate_{flag}) + \ln(rr_{tackle}).$$

We used Bayesian methods to estimate the parameters of the model given in equation [2]. A Bayesian approach was selected for a variety of reasons including flexibility of modeling random effects (Fong et al., 2010), the ability to estimate the uncertainty of functions of the model parameters, for example the probability of at least one impact per AE described below, and consistency with methods used to impute values for the missing AE counts for tackle players.

To complete specification of the model, we made the following non-informative prior distributional assumptions for the parameters in equation [2]

$rate_{flag} \sim N(0, 1000)$,

$rr_{tackle} \sim N(0, 1000)$, and

$\gamma \sim U(0,100)$.

Under the Poisson assumption on the impact counts, the probability that a player ij has X impacts during an AE is given by

$$P(Player\ ij\ has\ X\ Impacts) = \frac{e^{-\lambda_{ij}} {\lambda_{ij}}^{X}}{X!}.$$

Therefore, the probability that the player has at least one impact during an AE is

$$1 - P(Player\ ij\ has\ 0\ impacts) = 1 - e^{-\lambda_{ij}}\ .$$

Under the model in equation [2], the mean impact rate per AE among flag players is given by

$$\lambda_{flag} = e^{\ln(rate_{flag})}$$

while the mean impact rate for tackle players is

$$\lambda_{tackle} = e^{\ln(rate_{tackle})},$$

Therefore, estimates of the probabilities of at least one impact during a given AE are given by

$$p_{flag} = P(Flag\ player\ has\ at\ least\ one\ impact\ per\ AE) = 1 - e^{-\lambda_{flag}}$$

And

$$p_{tackle} = P(Tackle\ player\ has\ at\ least\ one\ impact\ per\ AE) = 1 - e^{-\lambda_{tackle}}\ .$$

Fitting the model given in equation [1] was complicated by the fact AE counts were not observed for tackle players. In the primary analyses, we addressed this missing data issue by setting AE_{ij} to a fixed value for each tackle player such that

$AE_{ij} = max_{ij}$ = the sum of the number of AEs in which any player on team j had at least one recorded impact.

This approach was assumed to be conservative in that using this definition for the missing tackle exposures likely leads to overestimation, on average, of the number of AEs attended by a player. This overestimation of the number of exposures tends to result in underestimation of the tackle impact rates. Note that AE data were observed for flag players and that this information was used in the analyses.

In an alternative approach, AE_{ij} for tackle players was treated as missing information and estimated simultaneously with the model parameters in the Bayesian updating process. The assumptions and the results of using this imputation approach are described below.

Model parameters were estimated using a Markov Chain Monte Carlo (MCMC) approach in which two chains with differing initial values were sampled. A total of 100,000 samples were selected from each chain with the initial 20,000 samples discarded to allow for convergence to the posterior distribution. In addition, only every 8th sample was retained in both chains to reduce potential autocorrelation. As a result, all estimates were based on 20,000 samples from the posterior distribution. Convergence was assessed using graphical comparison of the trace plots of the two sampling chains as well as convergence metrics such as the Gelman-Rubin statistic (Brooks and Gelman, 1998).

Uncertainty associated with the estimates is summarized using the 95% credible interval. This interval is defined using the 2.5th and 97.5th percentiles of the collection of 20,000 samples from the posterior distribution of each parameter. Under the assumptions associated with the models, there is a 95% probability that the unknown true values of the parameter lies within the limits of the presented credible intervals.

Estimation of Rates for Impacts ≥ 40 g

For this analysis, I_{ij} is redefined as total number of impacts of 40 g magnitude or higher, summed across all AEs, incurred by player i on team j. Preliminary analyses indicated that, for these higher magnitude impacts, the number of players with I_{ij} = 0 was greater than would be expected under a Poisson assumption for this random variable. To address this issue, we assume, for the higher magnitude impacts, that I_{ij} is sampled from a Zero Inflated Poisson (ZIP) (Ghosh et al., 2004) mixture distribution such that

$$I_{ij} = \begin{array}{l} 0 \text{ with probability } p_{zero} \\ I^*_{ij} \text{ with probability } (1 - p_{zero}) \end{array} \qquad [3]$$

Where

$$I^*_{ij} \sim Poisson(AE_{ij}\, \lambda_{ij}) .$$

The assumed model for I^*_{ij}, the Poisson component of the mixture model, is given in equation [2]. The parameter p_{zero} reflects the probability that the impact count is equal to zero in excess of the probability of a zero associated with the Poisson distribution. Prior assumptions for the parameters of the model for I^*_{ij} are identical to those made for modeling rates for impacts ≥ 10g with the addition of a uniform non-informative prior assumption for p_{zero} such that

$$p_{zero} \sim U(0,1).$$

The estimated probabilities of at least one impact ≥ 40 g per AE were developed using a modification of the estimators used in analyses of the ≥ 10 g data, To account for the increase in the probability of zero impacts, the probability of at least one impact of 40 g or higher per AE was modeled as

$$p_{flag} = 1 - \left(p_{zero} + e^{-\lambda_{flag}}\right)$$

for flag players and as

$$p_{tackle} = 1 - \left(p_{zero} + e^{-\lambda_{tackle}}\right)$$

for tackle.

Estimates of the rates of impacts ≥ 10 g and ≥ 40 g, tackle to flag rate ratios and probabilities of at least one impact per AE based on the models in equations [2] and [3] are provided in Table 1.

Table A1. Estimated impact rates[1], rate ratio and probability of at least one impact per athletic event for tackle and flag players.

Impact Magnitude	**Impact Rate (95% Credible Interval)**		**Tackle/Flag Rate Ratio** (95% Credible Interval)	**Probability ≥ 1 Impact** (95% Confidence Interval)	
	Type of Football			Type of Football	
	Flag	Tackle		Flag	Tackle
≥ 10 g[2]	0.63 (0.43,0.92)	9.19 (8.18,10.32)	14.67 (9.75,21.95)	0.47 (0.35,0.60)	1.00
≥ 40 g[3]	0.04 (0.03,0.07)	1.01 (0.90,1.13)	23.00 (13.59,39.55)	0.04 (0.03.0.07)	0.63 (0.58,0.67)

[1] Estimates of the impact rates were derived by setting the missing AE counts among tackle players to the number
of AEs in which any member of the player's team experienced a recorded impact.
[2] Estimates derived under the model given in equation [2].

[3] Estimates derived using the model given in equation [3].

Linear Acceleration

In our assessment of linear acceleration (LA), the outcomes of interest were the median and 95th percentile of each player's collection of observed LA measurements. Because evaluation of LA is conditional on the player having at least one impact in the course of the season, three flag players with zero total impacts across all AEs were excluded from this analysis.

We again used Bayesian methods to model the impact of playing tackle versus flag football in the assessment of linear acceleration. However, due to the LA measures being continuous outcomes, as opposed to impact counts, and the fact that several tackle and flag players had very large observed LA measures, we used a robust Bayesian approach to model these values (Williams and Martin, 2017). Methods used to identify potentially outlying LA measures and sensitivity assessment comparing the selected Student's t assumption to other approaches are described below.

Let la_{ij} be the linear acceleration measure, either the player's median value or 95th percentile, for player i on team j and assume

$$la_{ij} \sim t\big(\,\mu_{ij}\,, \sigma_k^2\,, df\big) \qquad [4]$$

where $t\big(\,\mu_{ij}\,, \sigma_k^2\,, df\big)$ is the Student's t distribution with mean μ_{ij}, variance $\sigma_k^2, k = 1{,}2$ and df degrees of freedom. The subscript, k, associated with the variance term reflects an assumption of differing values for this parameter for flag, $k = 1$ and tackle, $k = 2$, players.
To complete the model definition, we assume that

$$\mu_{ij} \;=\; la_{flag} \;+\; la\_diff_{tackle} * \; type_j \qquad [5]$$

Where

la_{flag}	= the mean of the LA measure among flag players,
la_diff_{tackle}	= the change in the mean LA measure due to playing tackle, and
$type_j$	= 1 if team j is tackle and 0 if team j is flag.

Note that, using the parameterization in equation [5], the mean of the LA measure for tackle players can be estimated as

$$la_{tackle} = la_{flag} + la\_diff_{tackle} \; .$$

Under the Bayesian approach, the degrees of freedom for the Student's t distribution is treated as another unknown random variable to be estimated in the updating process.

Prior assumptions for the model parameters in equation [5] were

$$la_{flag} \sim N(0, 1000\,)$$
$$la\_diff_{tackle} \sim N(0, 1000\,),$$
$$1/\sigma_k^2 \sim Gamma(0.001, 0.001) \text{ and}$$
$$df \sim U(2{,}100).$$

Estimates of the probability that the tackle LA average measure exceeds the associated flag value correspond to the percentage of samples from the posterior distribution in which the sampled tackle estimate exceeded that for flag.

Estimates of the population averages for the LA measures among tackle and flag players are provided in Table 2.

Table A2. Estimated average median and 95th percentile linear acceleration (LA) measures, increase in mean value due to playing tackle, degrees of freedom and probability that the tackle measure exceeds that among flag players.

Measure	**Mean LA** (95% Credible Interval) Type of Football		**Increase in Tackle Measure Compared to Flag**	**Degrees of Freedom**	**Probability Tackle ≥ Flag**
	Flag	Tackle			
Median	16.84 (15.57,18.21)	18.15 (17.95,18.34)	1.31 (-0.08,2.59)	3 (2,4)	0.97
95th Percentile	33.51 (28.23,39.08)	52.55 (51.06,54,09)	19.06 (13.38,24.45)	6 (4,12)	1.00

SENSITIVITY ASSESSMENTS

Comparison of Alternative Modeling Assumptions for Rates of Impacts ≥ 10 g

Alternative modeling assumptions were evaluated for comparison to results developed using the model given in equation [2]. These alternative assumptions included addition of team level random effects, consideration of a Negative Binomial, as opposed to Poisson, assumption on the likelihood of the observed impacts and evaluation of potential effects of player age division on impact rates. Comparison of models incorporating these alternative assumptions to the random intercept model given in equation [2] were based on examination of, when appropriate, the Deviation Information Criteria (DIC) (Lun et al., 2013), evaluation of various posterior predictive measures between those produced using the assumed models and the observed data (Berkof and van Mechelen, 2000) and changes in the estimates of interest, impact rates and probabilities of at least one impact, as a result of altering assumptions

The tables presented in this section provide summary information reflecting the results of fitting models for the rate of impacts ≥ 10 g under various assumptions. The goal is to compare the resulting estimates and model fit metrics to those associated with the estimates derived using the final model given in equation [2]. Four general areas are considered in the assessment: variation of assumptions on the hierarchical random effects included the model, evaluation of potential age division effects, examination of alternative distributional assumptions to address overdispersion in the impact data and, finally, an imputation-based sensitivity analysis focused on the uncertainty resulting from unobserved AE counts among tackle players.

Four sets of model comparison tables are presented to summarize the results of each of these assessments with each set comprised of three tables. The first table in each set lists model-based estimates for the impact rates, rate ratio and probability of at least one impact. In addition, the table contains the DIC when appropriate (lower DIC is better). The second two tables reflect posterior predictive checks for each model. The posterior predictive estimates associated with each model and can be thought of as predictions for a future hypothetical set of players. In this application, we compare these posterior predictions to the observed data to provide an indication of agreement between the model predictions and the observed values. The second table in each set shows the percent of posterior predictive impacts within Division that exceed the observed impact count. Ideally this value should be close to 50 percent. The third table provides an age division-specific comparison of the posterior predictive standard deviations, that is the standard deviation of the posterior predictive impact estimates generated using the model, to the corresponding standard deviations of the observed impacts. We consider this comparison important due to the substantial overdispersion of impact count values in these data. In this case, it is desirable for the predictive values to be close to observed.

In all comparisons below, the final model used for estimation of the rate of impacts ≥ 10 g, that is the model in equation [2], is denoted as the Common Player RE model reflecting an assumption that the player level random effects for both flag and tackle players are drawn from a common Normal distribution. To enable comparison, estimates produced using the Common Player RE model are highlighted in the tables.

Comparison of Poisson Models with Varying Assumptions on Hierarchical Random Effects

All models in the first set of comparisons are the based on the assumption that the impact counts follow a Poisson distribution with differing assumptions on the sub-models for overdispersion beyond that expected among Poisson random variables. For this section, exposure among tackle players is assumed to be known and is set to number of events in which any player on that player's team had at least one recorded impact.

The Poisson rate component for all models is given by:

$$\ln(\lambda_{ij}) = \ln(rate_{flag}) + \ln(rr_{tackle}) * type_j$$

with the parameters in the model as defined in equation [2].

Models evaluated for estimation of impact rates include:

Name	Model
No RE	No Random effects (RE) $\ln(\lambda_{ij}) = \ln(rate_{flag}) + \ln(rr_{tackle}) * type_j$
Common Player RE and tackle players	Player level random effects (RE) with common RE distribution for flag $\ln(\lambda_{ij}) = \ln(rate_{flag}) + \ln(rr_{tackle}) * type_j + u_{ij}$ $u_{ij} \sim N(0, \sigma^2)$
Common Team RE	Team level random effects with common RE distribution for flag and tackle players. $\ln(\lambda_{ij}) = \ln(rate_{flag}) + \ln(rr_{tackle}) * type_j + v_j$ $v_j \sim N(0, \delta^2)$
Separate Player RE	Player level random effects with separate RE distributions for flag and tackle players. $\ln(\lambda_{ij}) = \ln(rate_{flag}) + \ln(rr_{tackle}) * type_j + u_{ij}$ $u_{ij} \sim N(0, \sigma^2_{flag})$, if team j is flag

$u_{ij} \sim N(0, \sigma^2_{tackle})$, if team j is tackle.

Separate Team RE Team level random effects with separate RE distributions for flag and tackle players.

$$\ln(\lambda_{ij}) = \ln(rate_{flag}) + \ln(rr_{tackle}) * type_j + v_j$$

$v_j \sim N(0, \delta^2_{flag})$, if team j is flag

$v_j \sim N(0, \delta^2_{tackle})$, if team j is tackle

Common Team and Player RE Player and Team level random effects with common RE distributions for flag and tackle players.

$$\ln(\lambda_{ij}) = \ln(rate_{flag}) + \ln(rr_{tackle}) * type_j + u_{ij} + v_j$$

$u_{ij} \sim N(0, \sigma^2)$

$v_j \sim N(0, \delta^2)$

Separate Player RE by Age Division Player level random effects with separate RE distributions by age division.

$$\ln(\lambda_{ij}) = \ln(rate_{flag}) + \ln(rr_{tackle}) * type_j + u_{ik}$$

$u_{ik} \sim N(0, \sigma^2_k), k = 1,2,3,4,5,6,7,8$

k = Division

Priors for the parameters of the models above were identical to those assumed for the model in equation [2] with the addition of the following assumed prior distribution for the variance of the team level random effects, $\delta \sim U(0,100)$.

Estimated parameters of interest for the models considered in these analyses are presented in Table 3a while posterior predictive estimates are given in Tables 3b and 3c. Note that, comparing the Common Player RE model to models with only team level random effects, failure to include player level effects results in a substantial increase in the DIC and a tendency to underestimate the standard errors of the observed impact counts. Addition of team level effects to the Common Player RE model resulted in negligible improvement in fit as measured by the DIC. These results also indicate that assuming a common variance for the random effect distribution of tackle and flag players has little impact on both the estimates of interest and the posterior predictive checks when compared to more complex models based on assuming differing random effect distributions for flag and tackle players. Finally, assuming differing random effect distributions for each age division yields little evidence of a substantial improvement in model fit.

Table A3a. Estimated impact rates, rate ratios, probabilities of at least one impact per athletic event and Deviation Information Criteria (DIC) resulting from various assumptions on hierarchical player and team level random effects (RE).

Model	Impact Rate (95% Credible Interval)		Rate Ratio (95% Credible Interval)	Probability ≥ 1 Impact		DIC
	Type of Football			Type of Football		
	Flag	Tackle		Flag	Tackle	
No RE	1.08 (0.99,1.18)	14.88 (14.81,14.95)	13.74 (12.66,14.96)	0.66 (0.63, 0.69)	1.00	1.04E5
Common Player RE*	0.63 (0.43,0.92)	9.19 (8.18,10.32)	14.67 (9.75,21.95)	0.47 (0.35,0.60)	1.00	4694
Common Team RE	0.97 (0.71,1.34)	14.08 (12.40,15.94)	14.47 (10.27,20.37)	0.62 (0.51,0.74)	1.00	86160
Separate Player RE	0.65 (0.45,0.92)	9.20 (8,18,10.35)	14.11 (9.87,20.87)	0.48 (0.36,0.60)	1.00	4695
Separate Team RE	0.97 (0.53,1.69)	14.08 (12.48,15.89)	14.59 (8.32,26.85)	0.62 (0.41,0.81)	1.00	86160
Common Team and Player RE	0.97 (0.70,1.36)	14.06 (12.47,15.94)	14.44 (10.14,20.53)	0.62 (0.50,0.74)	1.00	86160
Separate Player RE by Age Division	0.76 (0.50,1.08)	9.80 (8.79,11.02)	12.90 (8.91,20.11)	0.53 (0.39,0.66)	1.00	4693

*Final model that was chosen.

Table A3b. Median percentage of posterior predictive impact counts that exceed the observed impact counts by age division for models based on various assumptions on hierarchical player and team level random effects (RE).

Model	Type of Football							
	Tackle				Flag			
	1	2	3	4	1	2	3	4
No RE	60	48	53	49	38	56	82	67
Common Player RE*	54	50	53	52	50	50	65	50
Common Team RE	56	48	53	52	63	56	65	50
Separate Player RE	54	50	53	52	50	50	65	50
Separate Team RE	56	48	53	52	63	63	65	50

Common Team and Player RE	56	48	53	52	63	56	65	50
Separate Player RE by Division	52	51	52	52	50	50	65	50

*Final model that was chosen.

Table A3c. Observed and median posterior predicted standard deviations of impact counts by age division based on various assumptions on hierarchical player and team level random effects (RE).

Model	**Type of Football**								**All**
	Tackle				Flag				
	1	2	3	4	1	2	3	4	
Observed	302	276	281	275	16	11	9	6	288
No RE	122	98	64	90	3	4	4	4	139
Common Player RE*	302	278	282	276	16	12	9	7	289
Common Team RE	162	162	142	109	4	6	3	3	175
Separate Player RE	302	278	282	276	16	12	8	7	289
Separate Team RE	160	161	141	108	5	5	3	2	175
Common Team and Player RE	162	162	142	109	4	6	3	3	175
Separate Player RE by Division	302	278	282	276	16	12	9	6	289

*Final model that was chosen.

Comparison of Poisson Models with and without Age Division Effects

The second set of model evaluation summaries reflect the potential effect of age division on the observed impact counts. Because the actual age of each player was not available in these data, we assigned the mid-point age associated with each players division as his age for this assessment. In this section, a linear model for the midpoint age in each division and a categorical age effect model, separated by ages < 10 and > 10, are fit with age effect modeled separately for flag and tackle players. Coefficients for the parameters reflecting the effect of mid-point age within division were modeled under the prior assumptions:

$\beta_1 \sim N(0,1000)$, and
$\beta_2 \sim N(0,1000)$.

As above, estimates produced using the model in equation [2] are presented for comparison in each table and are highlighted and referred to as the Common Player RE model.

Models considered in this section are:

<u>**Name**</u>	<u>**Model**</u>
Common Player RE	Player level random effects with common RE distribution for flag and tackle players $\ln(\lambda_{ij}) = \ln(rate_{flag}) + \ln(rr_{tackle}) * type_j + u_{ij}$ $u_{ij} \sim N(0, \sigma^2)$
Common Player RE Linear Age Effects	Team level random effect with separate linear age models fit for flag and tackle players. $\ln(\lambda_{ij}) = \ln(rate_{flag}) + \ln(rr_{tackle}) * type_j + u_{ij} +$ $\beta_1 * age_{ij} + \beta_2 * age_{ij} * type_j$ $u_{ij} \sim N(0, \sigma^2)$ age_{ij} = mid-point age of player $ij's$ Division
Common Player RE Categorical Age	Team level random effect with separate categorical age models fit for flag and tackle players. $\ln(\lambda_{ij}) = \ln(rate_{flag}) + \ln(rr_{tackle}) * type_j + u_{ij} +$

$$\beta_1 * \ age_{ij} + \ \beta_2 * age_{ij} * type_j$$

$$u_{ij} \sim N(0, \sigma^2)$$

$age_{ij} = 1$ if player ij > 10 years old, 0 otherwise.

To enable comparison of estimates across ages, estimates were produced using both the linear and categorical age models for players aged 8 and 12 years old.

The results of these assessments are summarized in Tables 4a, 4b and 4c. For tackle players, addition of age division effects had negligible impact on both the estimates of interest and the measures of model adequacy. Among flag players, the results presented in Table 4a indicate the potential for a decrease in impacts rates with increasing player age. However, due to the limited sample size among flag player older than 10 years, the similarity in DIC estimates for models containing and excluding age effects and the fact that the credible intervals for the estimated age parameters for flag players, β_1, contained zero, we did not include age division in the final models for flag impact rates.

Table A4a. Estimated impact rates at ages 8 and 12 and Deviation Information Criteria (DIC) produced using various assumptions on possible effects of player age.

Model	**Impact Rate at 8 Years** (95% Credible Interval)		**Impact Rate at 12 Years** (95 % Credible Interval)		DIC
	Type of Football		Type of Football;		
	Flag	Tackle	Flag	Tackle	
Common Player RE*^	0.63 (0.43,0.92)	9.19 (8.18,10.32)	0.63 (0.43,0.92)	9.19 (8.18,10.32)	4694
Common Player RE Linear Age Effect	0.97 (0.52,1.79)	9.37 (7.96,10.97)	0.46 (0.27,0.76)	9.06 (7.74,10.58)	4693
Common Player RE Categorical Age Effect[1]	0.97 (0.57,1.65)	10.36 (8.67,12.32)	0.38 (0.21,0.67)	8.43 (7.25,9.81)	4693

*Final model that was chosen.
^RE = Random effects.
[1] Age Categories defined as midpoint division age < 10 and midpoint division age > 10

Table A4b. Median percentage of posterior predictive impacts that exceed observed impact counts by age division produced using various assumptions on possible effects of player age.

Model	Type of Football							
	Tackle				Flag			
	1	2	3	4	1	2	3	4
Common Player RE*^	54	50	53	52	50	50	65	50
Common Player RE Linear Age Effect	54	50	53	52	50	50	65	50
Common Player RE Categorical Age Effect	54	51	53	52	50	56	59	50

*Final model that was chosen.
^RE = Random effects.

Table A4c. Observed and median posterior predictive standard deviations of impact counts by age division produced using various assumptions on possible effects of player age.

Model	Type of Football								All
	Tackle				Flag				
	1	2	3	4	1	2	3	4	
Observed	302	276	281	275	16	11	9	6	288
Common Player RE*^	302	278	282	276	16	12	9	7	289
Common Player RE Linear Age Effect	302	278	282	276	16	12	9	7	289
Common Player RE Categorical Age Effect	302	278	282	276	16	12	9	7	289

*Final model that was chosen.
^RE = Random effects.

Comparison of Poisson and Negative Binomial Models

In this section, the Poisson common player level random effects model is compared to models using a Negative Binomial assumption on impact counts both with and without player level random effects. DIC estimates for the Negative Binomial models are not available in the OpenBugs software that was used to fit these data (Lun et al., 2013). As a result, comparison of the effects of distributional assumptions on the resulting estimates of interest and posterior predictive assessment were used for model comparison.

Models considered in this section are:

<u>**Name**</u>	<u>**Model**</u>
Common Player RE	Player level random effects with common RE distribution for flag and tackle players. Impacts assumed to be Poisson random variables. $\ln(\lambda_{ij}) = \ln(rate_{flag}) + \ln(rr_{tackle}) * type_j + u_{ij}$ $u_{ij} \sim N(0, \sigma^2)$
NB No RE	No random effects. Impacts assumed to samples from a Negative Binomial distribution. $\ln(\lambda_{ij}) = \ln(rate_{flag}) + \ln(rr_{tackle}) * type_j$
NB Common Player RE	Player level random effects with common RE distribution for flag and tackle players. Impacts assumed to be samples from a Negative Binomial distribution. $\ln(\lambda_{ij}) = \ln(rate_{flag}) + \ln(rr_{tackle}) * type_j + u_{ij}$ $u_{ij} \sim N(0, \sigma^2)$

The results of these comparison are presented in Tables 5a, 5b and 5c. Note that use of Negative Binomial assumption for the distribution of impact counts resulted in an increase in the estimated values of the impact rates when compared to those developed under a Poisson assumption. However, the posterior predictive standard deviations of the impact counts produced using the Negative Binomial model were substantially larger that the observed impact standard deviations. In addition, the results in Table 5b suggest that use of the Negative Binomial assumption resulted in posterior predicted impact count estimates that were not centered close to the observe values. As a result, the Poisson assumption for the likelihood of the impact counts, as given in equation [2], was retrained for production of final estimates.

Table A5a. Estimated impact rates, rate ratios and probabilities of at least one impact per athletic event produced using models based on either a Poisson or Negative Binomial assumption on the likelihood of the observed impact counts.

Model	**Impact Rate** (95% Credible Interval)		**Rate Ratio**	**Probability ≥ 1 Impact**	
	Type of Football			Type of Football	
	Flag	Tackle		Flag	Tackle
Common Player RE*^	0.63 (0.43,0.92)	9.19 (8.18,10.32)	14.67 (9.75,21.95)	0.47 (0.35,0.60)	1.00
NB No RE	1.06 (0.80,1.46)	14.90 (13.62,16.33)	14.03 (10.08,19.0)	0.65 (0.55,0.77)	1.00
NB Common Player RE	1.06 (0.80,1.45)	14.89 (13.60,16.30)	14.0 (10.13,19.05)	0.65 (0.55,0.77)	1.00

*Final model that was chosen.
^RE = Random effects.

Table A5b. Median percentage of posterior predictive impacts that exceed the observed impact counts by age division produced using models based on either a Poisson or Negative Binomial assumption on the likelihood of the observed impact counts.

Model	**Type of Football**							
	Tackle				Flag			
	1	2	3	4	1	2	3	4
Common Player RE*^	54	50	53	52	50	50	65	50
NB No RE	52	42	49	45	39	44	71	50
NB Common Player RE	52	41	49	45	38	44	71	50

*Final model that was chosen.
^RE = Random effects.

Table A5c. Observed and median posterior predictive standard deviations of impact counts by age division produced using models based on either a Poisson or Negative Binomial assumption on the likelihood of the observed impact counts.

Model	**Type of Football**								**All**
	Tackle				Flag				
	1	2	3	4	1	2	3	4	
Observed	302	276	281	275	16	11	9	6	288
Common Player RE *^	302	278	282	276	16	12	9	7	289
NB No RE	387	389	420	400	10	12	10	9	404
NB Common Player RE	388	390	419	402	10	12	10	9	405

*Final model that was chosen.
^RE = Random effects.

Effect of Imputing Missing Tackle Exposure for Impacts ≥ 10 g

This section provides comparisons of estimates developed using an assumed value for each tackle player's missing AE to those developed treating this information as missing data to be imputed in the Bayesian estimation process. Because DIC estimates are not generally appropriate for evaluation of missing data models (Lun et al., 2013), these estimates are not presented for these comparisons. In this case, however, the models were not evaluated with model selection in mind as much as with the goal of assessing the sensitivity of the estimates to possible implications of not observing the true event exposures among tackle players.

The following models are evaluated in this section:

Name	**Imputation Model**
Common Player RE	Player level random effects with common RE distribution for flag and tackle players. Missing AE set to count of AEs in which any player on team has at least one recorded impact $\ln(\lambda_{ij}) = \ln(rate_{flag}) + \ln(rr_{tackle}) * type_j + u_{ij}$ $u_{ij} \sim N(0, \sigma^2)$
Uniform	Player level random effects with common RE distribution for flag and tackle players as in equation [2]. Tackle exposure imputed using a uniform distribution such that, $AE_{ij} \sim U(Min_{ij}, Max_{ij})$, when team j plays tackle = Observed AE_{ij}, when team j plays flag. Min_{ij} = Number of AEs in which player i had ≥ 1 impact. Max_{ij} = Number of AEs in which any member of team j has ≥ 1 impact

A comparison of the estimates and positive predictive metrics for these models is presented in Tables 6a, 6b and 6c. Note that imputation of the missing AE values tended, as expected, to produce a larger impact rate estimates among tackle players that that obtained by fixing the missing exposures at a likely conservatively large values. As a result, one could interpret the estimates produced using the imputation as a range of uncertainty for the true tackle impact rate reflecting lack of knowledge concerning the exposure among tackle players.

Table A6a. Estimated impact rates, rate ratios and probabilities of at least one impact per athletic event produced using assumed fixed values for the missing counts of athletic exposures among tackle players and using imputed values for these missing data.

Imputation Model	**Impact Rate** (95% Credible Interval)		**Rate Ratio**	**Probability ≥ 1 Impact**	
	Type of Football			Type of Football	
	Flag	Tackle		Flag	Tackle
Common Player RE*^	0.63 (0.43,0.91)	9.19 (8.18,10.32)	14.67 (9.75,21.95)	0.47 (0.35,0.60)	1.00
Uniform	0.68 (0.50,0.92)	15.87 (14.44,17.38)	23.32 (17.11,32.00)	0.49 (0.40,0.60)	1.00

*Final model that was chosen.
^RE = Random effects.

Table A6b. Median percentage of posterior predictive impacts that exceed observed impact counts by age division produced using assumed fixed values for the missing counts of athletic exposures among tackle players and using imputed values for these missing data.

Imputation Model	**Type of Football**							
	Tackle				Flag			
	1	2	3	4	1	2	3	4
Common Player RE*^	54	50	53	52	50	50	65	50
Uniform	54	51	53	52	50	50	71	50

*Final model that was chosen.
^RE = Random effects.

Table A6c. Observed and median posterior predictive standard deviations of impact counts by age division produced using assumed fixed values for the missing counts of athletic exposures among tackle players and using imputed values for these missing data.

Imputation Model	**Type of Football**								**All**
	Tackle				Flag				
	1	2	3	4	1	2	3	4	
Observed	302	276	281	275	16	11	9	6	288
Common Player RE*^	302	278	282	276	16	12	9	7	289
Uniform	302	278	282	276	16	12	8	7	289

*Final model that was chosen.
^RE = Random effects.

Comparison of Alternative Modeling Assumptions for Rates of Impacts ≥ 40 g

The impact of assuming various combinations of player and team level random effects, similar to that used to assess the model for ≥ 10g impacts in equation [2], was also assessed for the Poisson component, I_{ij}^{*}, of the model for ≥ 40 g impacts given in equation [3]. The results of these assessments were similar to those reported for ≥10 g impacts and indicated use of a model containing player level random effects for the Poisson component of the mixture model given in equation [3] provided a good description of the observed data. However, due to the substantial number of zero impact counts, especially among flag players, assessment of potential age effects was not addressed using a modeling process. Graphical assessment of the observed impact count and rate data, however, indicated no discernable effect of age division on impacts ≥ 40 g.

We did, however, evaluate alternative modeling assumptions for estimation of p_{zero} parameter. In particular, we evaluated the possibility that this variable may differ among flag and tackle players and the possibility that the probability of excess zero counts may differ by age division. Specifically, alternative models to that given in equation [3] evaluated were:

<u>**Name**</u>	<u>**Model**</u>
Common Player RE	Player level random effects with common RE distribution for flag and tackle players as in equation [2].
ZIP1	Player level random effects with common RE distribution for flag and tackle players as in equation [2]. For this model p_{zero} is assumed to be an unknown constant of equal value for both flag and tackle players. The prior distribution for p_{zero} was assumed to be $U(0,1)$. Note that the ZIP1 Model corresponds to the model in equation [3].
ZIP2	Player level random effects with common RE distribution for flag and tackle players as in equation [2]. For this model, p_{zero} is assumed to differ between flag and tackle players and is estimated using the logistic model $logit(p_{zero}) = \beta_0 + \beta_1 * type_j$, $type_j$ = 0 if team j is flag and 1 if it is tackle.
ZIP3	Player level random effects with common RE distribution for flag and tackle players as in equation [2]. For this model, p_{zero} is assumed to differ due to mid-point age of division and between flag and tackle

players. In this case, p_{zero} is estimated using the logistic model

$$logit(p_{zero}) = \beta_0 + \beta_1 * type_j + \beta_2 * age_{ij} + \beta_3 * age_{ij} * type_j$$

$type_j$ = 0 if team j is flag and 1 if it is tackle, and

age_{ij} = 1 if player ij > 10 years old, 0 otherwise.

Fitting the ZIP models relies on estimation of an unknown ordinal outcome, that is, the binary indicator of either a zero count or the assumed Poisson distribution. Due to reliance of the DIC estimates on approximate posterior Normality, DIC estimates are, again, not provided due to violation of this underlying assumption.

Results associated with the ZIP1 mode, that is the model given in equation [3], are highlighted in the tables below.

The estimates resulting from evaluation of alternative models for p_{zero} are presented in Table 7. Evaluation of these results indicates little impact of either type of football played or age division on the probability of having a zero-impact count. As a result, the ZIP1 model was used for p_{zero} in equation [3] for estimation of rates for impacts ≥ 40 g.

Table A7. Estimated rate of impacts ≥ 40 g for tackle and flag players, rate ratios, probabilities of at least one impact per athletic event and median posterior predictive count of zero impact counts produced using various models for the probability of excess zero impact counts.

Model	**Impact Rate** (95% Credible Interval)		**Rate Ratio**	**Probability ≥ 1 Impact**		**Median Estimated Zero Impacts**	
	Type of Football			Type of Football		Type of Football	
	Flag	Tackle		Flag	Tackle	Flag **(27[1])**	Tackle **(22)**
Common Player RE^	0.04 (0.02,0.07)	0.93 (0.82,1.05)	23.18 (13.33,42.10)	0.04 (0.02,0.07)	0.60 (0.56,0.65)	27	14
ZIP1*	0.04 (0.03,0.07)	1.01 (0.90,1.13)	23.00 (13.59,39.55)	0.04 (0.03,0.07)	0.63 (0.58,0.67)	27	24
ZIP2	0.04 (0.03,0.07)	1.01 (0.90,1.14)	23.23 (13.72,40.18)	0.04 (0.03,0.07)	0.63 (0.58,0.67)	27	23
ZIP3 Age < 10	0.04 (0.03,0.08)	1.02 (0.90,1.15)	23.10 (13.00,40.23)	0.04 (0.03,0.08)	0.64 (0.59,0.68)		
	0.04	0.98	23.10	0.04	0.61		

Age > 10	(0.02,0.07)	(0.87,1.10)	(13.54,42.35)	(0.02,0.07)	(0.57,0.65)	27	21

^RE = Random effects.
*Final model that was chosen.
[1] Bolded values within parentheses are the observed number of zero impact counts among tackle and flag players.

Effect of Imputing Tackle Exposure for Impacts ≥ 40 g

Missing AE values for tackle player were also imputed using the uniform model described above for ≥ 10 g impacts for impacts of 40 g magnitude or higher. The results of this imputation, along with a comparison to estimates produced using assumed fixed conservative values for AEs are presented in Table 8.

Table A8. Estimated impact rates, rate ratios and probabilities of at least one impact of magnitude ≥ 40 g per athletic event produced using assumed fixed values for the missing counts of athletic exposures among tackle players and one produced using imputed values for these missing data.

Imputation Model	**Impact Rate** (95% Credible Interval)		**Rate Ratio**	**Probability ≥ 1 Impact**	
	Flag	Tackle		Flag	Tackle
ZIP1[1*]	0.04 (0.03,0.07)	1.01 (0.90,1.13)	23.00 (13.59,39.55)	0.04 (0.03,0.07)	0.63 (0.58,0.67)
Uniform	0.05 (0,03, 0.08)	1.61 (1.44,1.80)	33.06 (20.28,55.07)	0.05 (0.03,0.08)	0.79 (0.75,0.82)

[1] No imputation was used in the ZIP1 model. As in the previous analyses, the missing AE values for each tackle
player was set to the number of AEs in which any member of that player's team had a recorded impact.
*Final model that was chosen.

Comparison of Alternative Modeling Assumptions for Linear Acceleration Measures

Potential outliers for the observed LA measures were identified as observations with values exceeding a robust cutoff, C. This value for C was derived separately for tackle and flag players and was set to the third quartile plus 2.5 time the interquartile range. Using this approach, nine LA values and three 95th percentile observed LA measures were identified as potential outliers. Sensitivity of the LA estimates produced using the model in equation [5] were examined when compared to estimates under an alternative, Normal, assumption for the likelihood as well as for the impact of inclusion and exclusion of the identified outliers. In addition, potential impacts of player age division were also examined as was the impact of inclusion of team level random effects in the model. Models including player-level random effects were not evaluated due to identifiability issues resulting from the inability to separate the estimated variance of the random effects from the estimated variance of the observations about the modeled mean.

Letting, la_{ij} be the linear acceleration value of interest, median or 95th percentile, the models considered were:

<u>**Name**</u>	<u>**Model**</u>
Normal	Normal likelihood with no random effects.
Outliers Included	$la_{ij} \sim N(u_{ij}, \sigma_j^2)$
No Team RE	$\mu_{ij} = la_{flag} + la\_diff_{tackle} * type_j$
	Excluded observations: 5 players with questionable impact counts.
Normal	Normal likelihood with team-level random effects.
Outliers Included	$la_{ij} \sim N(u_{ij}, \sigma_j^2)$
Team RE	$\mu_{ij} = la_{flag} + la\_diff_{tackle} * type_j + v_j \, , \; v_j \sim N(0, \delta^2)$.
	Excluded observations: 5 players with questionable impact counts.
Normal	Normal likelihood with no random effects
Outliers Excluded	$la_{ij} \sim N(u_{ij}, \sigma_j^2)$
No Team RE	$\mu_{ij} = la_{flag} + la\_diff_{tackle} * type_j$
	Excluded observations: Identified LA outliers, 5 players with

	questionable impact counts.
Student's t	Student's t likelihood with team-level random effects.
Outliers Included	$la_{ij} \sim t(\mu_{ij}, \sigma^2, df)$
Team RE	$\mu_{ij} = la_{flag} + la\_diff_{tackle} * type_j + v_j, , v_j \sim N(0, \delta^2)$.
	Excluded observations: 5 players with questionable impact counts.
Student's t	Student's t likelihood with no random effects. Note, this is the
Outliers Included	given in equation [4].
No Team RE	$la_{ij} \sim t(\mu_{ij}, \sigma^2, df)$
	$\mu_{ij} = la_{flag} + la\_diff_{tackle} * type_j$
	Excluded observations: 5 players with questionable impact counts.
Student's t	Student's t likelihood with team-level random effects.
Outliers Excluded	$la_{ij} \sim t(\mu_{ij}, \sigma^2, df)$
Team RE	$\mu_{ij} = la_{flag} + la\_diff_{tackle} * type_j + v_j, , v_j \sim N(0, \delta^2)$.
	Excluded observations: Identified LA outliers, 5 players with questionable impact counts.
Student's t	Student's t likelihood with team-level random effects.
All Data	$la_{ij} \sim t(\mu_{ij}, \sigma^2, df)$
Team RE	$\mu_{ij} = la_{flag} + la\_diff_{tackle} * type_j + v_j, , v_j \sim N(0, \delta^2)$.
	Excluded observations: None

The following prior assumptions were used for all models:

$la_{flag} \sim N(0, 1000)$,

$la\_diff_{tackle} \sim N(0, 1000)$,

$1/_{\sigma^2} \sim Gamma(0.001, 0.001)$,

$\delta \sim U(0,100)$, and

$$df \sim U(2{,}100)\ .$$

The results of these sensitivity assessments are provided in Tables 9a, for the median LA, and in Table 9b for the 95th percentile. Note that, for both outcomes, estimates for the average LA measures under the assumed Student's t likelihood with the potential outliers included were close to those produced when the outliers were excluded. This result is indicative of the robustness of the approach to highly dispersed data. In addition, use of an assumed Normal likelihood for the LA measures resulted in a substantial increase in the DIC measure as compared to the fit produced using a Student's t assumption. Recalling, that a lower DIC is, in general, indicative of a model in greater agreement with the data, this provides additional evidence for the adequacy of the Student's t distributional assumption. Alternatively, while inclusion of the team level random effects did result in a lower DIC estimates, addition of these terms had virtually no impact in the estimates of the average LA measures. As a result, final estimates were developed using the less complex model without team level random effects. Finally, as shown in the last row of Tables 8a and 8b, exclusion of the 5 players with questionable impact counts had no effect on the resulting estimates of interest in the LA analyses.

Table A9a. Estimated average median linear acceleration (LA), increase in mean value due to playing tackle, probability that tackle measure exceeds that among flag players, degrees of freedom (df) and Deviance Information Criteria (DIC) under various modeling assumptions.

Model	Mean LA (95% Credible Interval) Type of Football		Tackle Increase in LA	Probability Tackle LA ≥ Flag LA	df	DIC
	Flag	Tackle				
Normal - Outliers Included No Team RE^	18.76 (16.48,21.08)	18.73 (18.30, 19.15)	-0.04 (-2.38,2.27)	0.49		3134
Normal - Outliers Included Team RE	18.75 (16.39,21.03)	18.73 (18.30,19.15)	-0.03 (-2.35,2.35)	0.49		3134
Normal - Outliers Excluded No Team RE	17.43 (16.17,18.71)	18.37 (18.18,18.57)	0.94 (-0.35,2.22)	0.92		2308
Student's t - Outliers Included Team RE	16.80 (15.41,18.29)	18.14 (17.80,18.47)	1.34 (-0.17,2.76)	0.96	3 (2, 3)	2411
Student's t - Outliers Included No Team RE*	16.84 (15.57,18.21)	18.15 (17.95,18.34)	1.31 (-0.08,2.59)	0.97	3 (2,4)	2460
Student's t - Outliers Removed Team RE	16.98 (15.66,18.38)	18.20 (17.88,18.51)	1.22 (-0.20,2.58)	0.95	6 (4,14)	2255
Student's t - All Data Team RE	16.97 (15.65, 18.36)	18.13 (17.80,18.47)	1.17 (-0.27,2.52)	0.95	3 (2,4)	2434

^RE = random effect.
*Final model that was chosen.

Table A9b. Estimated average 95th percentile linear acceleration (LA), increase in mean value due to playing tackle, probability that tackle measure exceeds that among flag players, degrees of freedom (df) and Deviance Information Criteria (DIC) under various modeling assumptions.

Model	**Mean LA** (95% Credible Interval) Type of Football		**Tackle Increase in LA**	**Probability Tackle LA ≥ Flag LA**	***df***	**DIC**
	Flag	Tackle				
Normal - Outliers Included No Team RE^	35.61 (29.77,41.54)	53.68 (52.04,55.30)	18.03 (11.88,24.12)	1.00		4505
Normal - Outliers Included Team RE	35.62 (29.59,41.40)	53.66 (52.02,55.30)	18.05 (12.05,24.27)	1.00		4505
Normal - Outliers Excluded No Team RE	33.85 (29.14,38.61)	53.27 (51.74,54.83)	19.42 (14.47,24.40)	1.00		4400
Student's t - Outliers Included Team RE	33.26 (26.26,40.45)	51.99 (49.53,54.42)	18.75 (11.11,26.11)	1.00	5 (3,7)	4426
Student's t - Outliers Included No Team RE*	33.51 (28.23,39.08)	52.55 (51.06,54.09)	19.06 (13.38,24.45)	1.00	6 (4,12)	4465
Student's t - Outliers Removed Team RE	33.35 (26.57,39.88)	52.29 (49.86,54.67)	18.94 (11.96,26.02)	1.00	8 (4,60)	4365
Student's t - All Data Team Re	33.92 (26.64,41.16)	51.98 (49.52,54.42)	18.06 (10.49,25.78)	1.00	4 (2,7)	4483

^RE = random effect.
*Final model that was chosen.

Comparison of Student's t Likelihood models for linear acceleration with and without age division effects

The model for LA given in equation [5] was modified to enable evaluation of the potential impact of player age division on the average LA measure estimates. As with the previous analyses, age was assigned to each player using the mid-point age of his age division. Again, a linear model for the midpoint age and a categorical age effect model, separated by ages < 10 and > 10, were fit with age effects modeled separately for flag and tackle players. As in earlier analyses, estimates are developed for ages 8 and 12 years to enable comparison of modeling assumptions. The Student's t model with no age effects given in equations [4] and [5] is highlighted in the tables below.

Estimates for the average LA measures and DIC are provided in Tables 10a and 10b. Note that there is little difference between the estimates when models did or did not include age effects under both the linear and categorical age models. In addition, the 95% credible intervals associated with the estimates have considerable overlap and the addition of age effects to the model had little impact on the estimated DIC. As a result, age effects were not included in the final models for estimating the average LA measures.

Table A10a. Estimated median linear acceleration (LA) estimates and deviation information criteria (DIC) based on a Student's t likelihood model with no age division effects and for models with linear and categorical age effects.

Model	**Average LA at 8 Years** (95% Cred. Int.)		**Average LA at 12 Years** (95 % Cred. Int.)		DIC
	Type of Football		Type of Football		
	Flag	Tackle	Flag	Tackle	
Students' t*[1]	16.84 (15.57,18.21)	18.15 (17.95,18.34)	16.84 (15.57,18.21)	18.15 (17.95,18.34)	2460
Student's t Linear Age Effect	15.99 (14.15,17.98)	18.39 (18.11,18.67)	17.55 (15.91,19.44)	19.94 (17.42,22.70)	2457
Students' t Categorical Age Effect[2]	16.40 (14.72,18.23)	18.41 (18.11,18.71)	17.46 (15.62,19.40)	17.98 (17.73,18.22)	2459

*Final model that was chosen.
[1] This model corresponds to that given in equations [4] and [5]
[2] Age categories defined as midpoint Division age < 10 and midpoint division age > 10

Table A10b. Estimated 95th percentile median linear acceleration (LA) estimates and deviation information criteria (DIC) based on a Student's t likelihood model with no age division effects and for models with linear and categorical age effects.

Model	Average LA at 8 Years (95% Credible Interval)		Average LA at 12 Years (95 % Credible Interval)		DIC
	Type of Football		Type of Football		
	Flag	Tackle	Flag	Tackle	
Students' t*[1]	33.51 (28.23,39.08)	52.55 (51.06,54.09)	33.51 (28.23,39.08)	52.55 (51.06,54.09)	4465
Student's t Linear Age Effect	36.32 (27.01,45.62)	53.09 (50.95,55.22)	31.53 (24.36,38.65)	48.28 (35.67,60.81)	4468
Students' t Categorical Age Effect[2]	36.35 (29.10,43.64)	53.01 (50.74,55.29)	30.45 (23.21,38.15)	52.19 (50.26,54.23)	4467

*Final model that was chosen.
[1] This model corresponds to that given in equations [4] and [5].
[2] Age categories defined as midpoint Division age < 10 and midpoint division age > 10

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